

# Emergence of Lowenstein-Zimmermann mass terms for $\text{QED}_3$

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In this paper we consider a super-renormalizable theory of massless QED in  $(2+1)$  dimensions and discuss their BRST symmetry transformation. By extending the BRST transformation we derive the Nielsen identities for the theory. Further, we compute the generalized BRST (so-called FFBRST) transformation by making the transformation parameter field-dependent. Remarkably, we observe that the Lowenstein-Zimmerman mass terms, containing Lowenstein-Zimmerman parameter which plays an important role in the BPHZL renormalization program, along with the external sources coupled to the non-linear BRST variations appear naturally in the theory through a FFBRST transformation.

*Keywords:*  $\text{QED}_3$ ; Lowenstein-Zimmermann mass; BRST symmetry.

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## I. INTRODUCTION

Massless QED in  $2 + 1$  dimensions ( $\text{QED}_3$ ) has very interesting and crucial features in the frontier research. Recently in a seminal work [1],  $\text{QED}_3$  has been established a super-renormalizable theory utilizing a powerful algebraic renormalization method. Also, it is a ultraviolet finite and parity invariant theory [1–5]. The massless  $\text{QED}_3$  provides an ideal platform to tackle the infrared divergence present in the theory. The parity anomaly has been dismissed for such theories at all orders of perturbation. The Lowenstein-Zimmerman scheme plays an important role in algebraic proof of ultraviolet and infrared finiteness and in the dismissal of parity anomaly. In spite of that the  $\text{QED}_3$  also gets relevance in the study of high- $T_c$  superconductivity [6, 7]. The dynamical fermion mass generation and chiral symmetry breaking for  $\text{QED}_3$  are studied in [8]. In this context, the dynamical mass generation using Hamiltonian lattice methods has also been investigated which has been found in agreement with both the strong coupling expansion and with the Euclidean lattice simulations [9].

The Lowenstein-Zimmermann subtraction scheme plays a major role in the algebraic proof on the ultraviolet and infrared finiteness, and to show the absence of a parity and infrared anomaly, in the massless  $\text{QED}_3$ , which is based on general theorems of perturbative quantum field theory [10–18]. The massless  $\text{QED}_3$  with and without Lowenstein-Zimmermann mass terms is a gauge theory and in order to deal such theories we need to break the gauge invariance by fixing the gauge. The gauge invariance of the quantum action is also essential because the expectation values of physical quantities become independent of the choice of the gauge-fixing term. To realize the gauge independence i.e. the estimating S-matrix elements (or expectation values) of the gauge independent quantities, one utilizes the on-shell quantum effective action (i.e., evaluated at those configurations that extremize it). Nielsen identities [19] suggest that the variation of the quantum effective action due to changes in the functions that fix the gauge must be linear in the quantum corrected equations of motion for the mean fields. This follows that the on-shell quantum effective action does not depend on the choice of the gauge breaking term. Although the mean fields depend on the gauge-fixing, this dependence gets canceled by the explicit gauge-fixing dependence of the quantum effective action [20–22].

In the proof of algebraic renormalizability the BRST symmetry has an incredible importance [23]. The generalizations of BRST symmetry have been studied in various contexts [24–52] since their introduction

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in Ref. [24]. For examples, a correct prescription for poles in the gauge field propagators in noncovariant gauges has been derived by connecting covariant gauges and noncovariant gauges of the theory by using FFBRST transformation [29]. The long outstanding problem of divergent energy integrals in Coulomb gauge has been regularized using FFBRST transformation [26]. The Gribov-Zwanziger theory [53, 54], a limiting case of Yang-Mills theory, which plays a crucial role in the non-perturbative low-energy region while it can be neglected in the perturbative high-energy region, has also been related to the YM theory in Euclidean space through FFBRST transformation [55, 56]. The FFBRST formulation has also been established at quantum level utilizing BV formulation [25]. Recently, the field-dependent BRST transformation has also been considered with same philosophy and goal as in original work [24] even though in slightly different manner to calculate the explicit Jacobian for such transformation [57–60]. A systematic Hamiltonian formulation of such theories have also been done [61, 62].

In this paper we analyse the massless QED<sub>3</sub>, which is a super-renormalizable and free from parity and infrared anomaly, within generalized BRST (FFBRST) framework. To implement the FFBRST formulation, we first make the infinitesimal parameter of transformation field dependent through continuous interpolation of a parameter  $\kappa : (0 \leq \kappa \leq 1)$ . Then we integrate the infinitesimal field-dependent transformation parameter with respect to  $\kappa$  in its extreme limit to obtain the FFBRST transformation. Such FFBRST transformation leads a non-trivial Jacobian for the path integral measure of functional integral. Remarkably, we realize that the Lowenstein-Zimmerman mass terms along with the external sources for QED<sub>3</sub> emerges naturally through Jacobian calculation under FFBRST transformation. In proof of renormalizability of the theory such source term play an important role.

The plan of the paper is as follows. We start with a brief discussion of massless QED<sub>3</sub> theory in Sec. II. The section III is devoted to derive the Nielsen identities. Further, in section IV, we sketch the generalized BRST transformation. In Sec. V, we show the emergence of Lowenstein-Zimmermann mass terms along with external source term for QED<sub>3</sub>. The last section is reserved for conclusions.

## II. THE MASSLESS QED<sub>3</sub>

In this section, we recapitulate the massless QED<sub>3</sub> with and without Lowenstein-Zimmermann mass terms. Let us begin with the gauge invariant massless action for QED<sub>3</sub> defined as

$$\Sigma_0 = \int d^3x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} \gamma^\mu D_\mu \psi \right], \quad (1)$$

where the covariant derivative is defined by  $D_\mu = \partial_\mu + ieA_\mu$  and  $e$  is a dimensionful coupling constant. Since, the gauge invariant theory can be quantized correctly only after choosing a particular gauge for the theory. We define the gauge-fixing and induce ghost terms for QED<sub>3</sub> as follows:

$$\Sigma_{gf+gh} = \int d^3x \left[ b\partial^\mu A_\mu + \frac{\xi}{2} b^2 + \bar{c}\partial_\mu \partial^\mu c \right], \quad (2)$$

where  $b, c$  and  $\bar{c}$  are Nakanishi-Lautrup auxiliary field, ghost field and anti-ghost field respectively. Therefore, the effective action can be written easily by

$$\Sigma_{eff} = \Sigma_0 + \Sigma_{gf+gh}. \quad (3)$$

This quantum action remains invariant under following nilpotent BRST transformations:

$$\begin{aligned} \delta_b A_\mu &= s_b A_\mu \epsilon = \frac{1}{e} \partial_\mu c \epsilon, \\ \delta_b \psi &= s_b \psi \epsilon = ic\psi \epsilon, \\ \delta_b \bar{\psi} &= s_b \bar{\psi} \epsilon = -i\bar{\psi} c \epsilon, \\ \delta_b c &= s_b c \epsilon = 0, \\ \delta_b \bar{c} &= s_b \bar{c} \epsilon = \frac{1}{e} b \epsilon, \\ \delta_b b &= s_b b \epsilon = 0, \end{aligned} \quad (4)$$

where  $\epsilon$  is an infinitesimal anticommuting parameter of transformation.

Now, we introduce a gauge invariant Lowenstein-Zimmermann mass term for the massless QED<sub>3</sub> which makes the theory super-renormalizable [1]:

$$\Sigma_m = \int d^3x \left[ \frac{\mu}{2}(s-1)\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + m(s-1)\bar{\psi}\psi \right], \quad (5)$$

where  $0 \leq s \leq 1$  is the Lowenstein-Zimmermann parameter which plays a central role in BPHZL renormalization program. Now, the effective action for QED<sub>3</sub> with the Lowenstein-Zimmermann mass terms is given by

$$\Sigma_{eff}^m = \Sigma_{eff} + \Sigma_m. \quad (6)$$

It is desirable to define an external source term for the theory [1],

$$\Sigma_{ext} = \int d^3x [\bar{\Omega}s_b\psi - s_b\bar{\psi}\Omega]. \quad (7)$$

This source terms play an important role in demonstrating the Slavnov-Taylor identity which guarantees the renormalizability of the theory. With this source term the final action reads,

$$\Sigma_T = \Sigma_{eff} + \Sigma_m + \Sigma_{ext}. \quad (8)$$

This final action ( $\Sigma_T$ ) is invariant under the same set of BRST transformations (4). Now we would like to derive the Nielsen identities for the QED<sub>3</sub> with the Lowenstein-Zimmermann mass terms in order to discuss the gauge independence of the physical quantity.

### III. NIELSEN IDENTITIES

To study the Nielsen identities we extend the action by introducing a global Grassmannian variable,  $\chi$  as follows

$$\Sigma_{eff}^{m'} = \Sigma_{eff}^m + \int d^3x \frac{\chi}{2}\bar{c}b. \quad (9)$$

Here we remark that the addition of this term does not change the dynamics of the theory because of the Grassmannian nature of  $\chi$ . This extended action (9) remains unchanged under the following set of extended BRST transformations:

$$\begin{aligned} \delta_b^+ A_\mu &= \frac{1}{e}\partial_\mu c \epsilon, & \delta_b^+ \psi &= ic\psi \epsilon, \\ \delta_b^+ \bar{\psi} &= -ic\bar{\psi} \epsilon, & \delta_b^+ c &= 0, \\ \delta_b^+ \bar{c} &= \frac{1}{e}b \epsilon, & \delta_b^+ b &= 0 \\ \delta_b^+ \xi &= -\frac{1}{e}\chi \epsilon, & \delta_b^+ \chi &= 0, \end{aligned} \quad (10)$$

where  $\epsilon$  is a transformation parameter. Now, to derive the Nielsen identities we define the following path integral with sources

$$Z = \int \mathcal{D}\phi \exp i \left[ \Sigma_{eff}^m + \int d^3x (J_\mu A^\mu + \bar{J}_\psi \psi + \bar{\psi} J_{\bar{\psi}} + bJ_b + \bar{\Omega}(-ic\psi) + ic\bar{\psi}\Omega) \right], \quad (11)$$

where various  $J$  are the sources respective to associated fields and composite fields. Now we define the vertex functional as follows

$$\Delta(A_\mu, \psi, \bar{\psi}, b, c, \bar{c}, \chi, \xi, \bar{\Omega}, \Omega) = W(J_\mu, J_{\bar{\psi}}, \bar{J}_\psi, J_b, \Omega, \bar{\Omega}, \chi, \xi) - \int d^3x (J_\mu A^\mu + \bar{J}_\psi \psi + \bar{\psi} J_{\bar{\psi}} + bJ_b), \quad (12)$$

where the generating functional  $W$  generates only connected Green's function. To study the gauge dependence of the propagators, we now introduce the functional integral of proper Green functions

$$\delta_b^+ A_\mu \frac{\delta \Delta}{\delta A_\mu} + \delta_b^+ \psi \frac{\delta \Delta}{\delta \psi} + \delta_b^+ \bar{\psi} \frac{\delta \Delta}{\delta \bar{\psi}} + \delta_b^+ \bar{c} \frac{\delta \Delta}{\delta \bar{c}} + \delta_b^+ \xi \frac{\delta \Delta}{\delta \xi} = 0. \quad (13)$$

Now we demand the invariance of the above functional under the extended BRST transformations which yields

$$\frac{1}{e} \partial_\mu c \frac{\delta \Delta}{\delta A_\mu} + \frac{\delta \Delta}{\delta \Omega} \frac{\delta \Delta}{\delta \psi} + \frac{\delta \Delta}{\delta \bar{\Omega}} \frac{\delta \Delta}{\delta \bar{\psi}} + \frac{1}{e} b \frac{\delta \Delta}{\delta \bar{c}} - \chi \frac{\delta \Delta}{\delta \xi} = 0. \quad (14)$$

Differentiation of above equation with respect to  $\chi$  and then set  $\chi$  equal to zero results

$$\frac{\delta \Delta}{\delta \xi} + \frac{1}{e} \partial_\mu c \frac{\delta^2 \Delta}{\delta \chi \delta A_\mu} - \frac{\delta^2 \Delta}{\delta \chi \delta \bar{\Omega}} \frac{\delta \Delta}{\delta \bar{\psi}} + \frac{\delta \Delta}{\delta \bar{\Omega}} \frac{\delta^2 \Delta}{\delta \chi \delta \bar{\psi}} - \frac{\delta^2 \Delta}{\delta \chi \delta \Omega} \frac{\delta \Delta}{\delta \psi} + \frac{\delta \Delta}{\delta \Omega} \frac{\delta^2 \Delta}{\delta \chi \delta \psi} - \frac{1}{e} b \frac{\delta^2 \Delta}{\delta \chi \delta \bar{c}} = 0. \quad (15)$$

This is the most general expression for the Nielsen identities for the QED<sub>3</sub> with the Lowenstein-Zimmermann mass terms. From these expression we can generate the Nielsen identities for the two-point functions. With this result one can show that the pole mass of the electron is gauge independent and that the photon self-energy can be simply shown to be gauge parameter independent.

#### IV. FFBRST TRANSFORMATION

Let us review the FFBRST formulation [24] in brief. To do so, we first write the usual BRST transformation for a generic field  $\phi$  written collectively for massless QED<sub>3</sub> theory,

$$\delta_b \phi = s_b \phi \epsilon = \mathcal{R}[\phi] \epsilon, \quad (16)$$

where  $\mathcal{R}[\phi] = s_b \phi$  is Slavnov variation of  $\phi$  and  $\epsilon$  is infinitesimal parameter of transformation. The importance of the BRST transformation does not alter by considering (i) the finite or infinitesimal and (ii) the field-dependent or field-independent versions of the parameter  $\delta \Lambda$  provided the parameter must be anticommuting and space-time independent. This observation gives us a freedom to generalize the BRST transformation by making the parameter,  $\epsilon$ , finite and field-dependent. We first define the infinitesimal field-dependent transformation as follows [24]

$$\frac{d\phi(x, \kappa)}{d\kappa} = \mathcal{R}[\phi(x, \kappa)] \Theta'[\phi(x, \kappa)], \quad (17)$$

where the  $\Theta'[\phi(x, \kappa)]$  is an infinitesimal field-dependent parameter. The FFBRST transformation ( $\delta_f$ ) with the finite field-dependent parameter then can be obtained by integrating the above transformation from  $\kappa = 0$  to  $\kappa = 1$ , as follows:

$$\delta_f \phi(x) \equiv \phi(x, \kappa = 1) - \phi(x, \kappa = 0) = \mathcal{R}[\phi(x)] \Theta[\phi(x)], \quad (18)$$

where  $\Theta[\phi(x)]$  is the finite field-dependent parameter constructed from its infinitesimal version. Under such FFBRST transformation with finite field-dependent parameter the measure of generating function will not be invariant and will contribute some non-trivial terms to the generating function in general [24].

The Jacobian of the path integral measure ( $\mathcal{D}\phi$ ) in the functional integral for such transformations is then evaluated for some particular choices of the finite field-dependent parameter,  $\Theta[\phi(x)]$ , as follows

$$\mathcal{D}\phi' = J(\kappa) \mathcal{D}\phi(\kappa). \quad (19)$$

Now, we replace the Jacobian  $J(\kappa)$  of the path integral measure as

$$J(\kappa) \longmapsto e^{i\Sigma_1[\phi(x, \kappa)]}, \quad (20)$$

iff the following condition [24]

$$\int \mathcal{D}\phi(x) \left[ \frac{d}{d\kappa} \ln J(\kappa) - i \frac{d\Sigma_1[\phi(x, \kappa)]}{d\kappa} \right] \exp[i(\Sigma_{eff} + \Sigma_1)] = 0 \quad (21)$$

is satisfied where  $\Sigma_1[\phi]$  is some local functional of fields satisfying initial boundary condition  $\Sigma_1[\phi]|_{\kappa=0} = 0$ .

Moreover, the infinitesimal change in Jacobian,  $J(\kappa)$ , is calculated by [24]

$$\frac{d}{d\kappa} \ln J(\kappa) = - \int d^3y \left[ \pm \sum_i \mathcal{R}[\phi^i(y)] \frac{\partial \Theta'[\phi(y, \kappa)]}{\partial \phi^i(y, \kappa)} \right], \quad (22)$$

where, for bosonic fields, + sign is used and for fermionic fields, - sign is used.

Therefore, by constructing an appropriate  $\Theta$ , we can calculate the non-trivial (local) Jacobian which extends the effective action by a term  $\Sigma_1$ .

## V. EMERGENCE OF LOWENSTEIN-ZIMMERMANN MASS TERMS

In this section, we explicitly show the emergence of Lowenstein-Zimmermann mass terms for the massless QED<sub>3</sub> theory under FFBRST formulation. To implement these notions, we construct the FFBRST transformation corresponding to Eq. (4) following the techniques outlined in Sec. III:

$$\begin{aligned} \delta_f A_\mu &= \frac{1}{e} \partial_\mu c \Theta[\phi], \\ \delta_f \psi &= ic\psi \Theta[\phi], \\ \delta_f \bar{\psi} &= -ic\bar{\psi} \Theta[\phi], \\ \delta_f c &= 0, \\ \delta_f \bar{c} &= \frac{1}{e} b \Theta[\phi], \\ \delta_f b &= 0, \end{aligned} \quad (23)$$

where  $\Theta[\phi]$  is an arbitrary field-dependent parameter of transformation. It is easy to check that the effective action for QED<sub>3</sub> given in (8) is invariant under these set of transformations. Now, we construct a particular field-dependent parameter (following the procedure given in Ref. [24]) as

$$\Theta[\phi] = \int d^3x e^{\frac{\bar{c}b}{b^2}} \left[ \exp \left( i \frac{\mu}{2} (s-1) \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + im(s-1) \bar{\psi}\psi + i\psi\bar{\Omega} - i\bar{\psi}\Omega \right) - 1 \right]. \quad (24)$$

Now, following the method discussed in section III, we calculate the Jacobian for path integral measure under finite field-dependent BRST transformation with above parameter as follows

$$J[\phi(x)] = e^{i\Sigma_1} = e^{i \int d^3x \left[ \frac{\mu}{2} (s-1) \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + m(s-1) \bar{\psi}\psi + \bar{\Omega} s_b \psi - s_b \bar{\psi} \Omega \right]}. \quad (25)$$

Here to obtain this expression we have utilized the relation (21).

As a consequence of performing FFBRST transformation on path integral measure of functional integral we see that the effective action of the theory gets extended (within functional integral) as follows:

$$\begin{aligned} \Sigma_{eff} + \Sigma_1 &= \int d^3x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} D\psi + b\partial^\mu A_\mu + \frac{\xi}{2} b^2 + \bar{c}\partial_\mu \partial^\mu c \right. \\ &\quad \left. + \frac{\mu}{2} (s-1) \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + m(s-1) \bar{\psi}\psi + \bar{\Omega} s_b \psi - s_b \bar{\psi} \Omega \right]. \end{aligned} \quad (26)$$

which exactly coincides with effective action given in (8). This justifies our claim of emergence of Lowenstein-Zimmermann mass terms naturally under the celebrated FFBRST technique. We also emphasized that the external source terms for the non-linear BRST variations which are required to prove the renormalizability of the theory are automatically generated through the same FFBRST transformation.

It means that under FFBRST transformation with appropriately constructed parameter (24) the effective action (within functional integral) changes as

$$\Sigma_{eff} + \Sigma_1 = \Sigma_T. \quad (27)$$

Hence, the whole mechanism is, precisely, given by

$$\int \mathcal{D}\phi e^{i\Sigma_{eff}} \xrightarrow{\text{FFBRST}} \int \mathcal{D}\phi e^{i(\Sigma_{eff} + \Sigma_1)} = \int \mathcal{D}\phi e^{i(\Sigma_{eff} + \Sigma_m + \Sigma_{ext})}, \quad (28)$$

where the generic path integral measure ( $\mathcal{D}\phi$ ) is explicitly given by  $\mathcal{D}\phi = \mathcal{D}A_\mu \mathcal{D}b \mathcal{D}c \mathcal{D}\bar{c} \mathcal{D}\psi \mathcal{D}\bar{\psi}$ . Therefore, the Lowenstein-Zimmermann mass terms and the external source term are generated through the Jacobian of the path integral measure under generalized BRST transformations with appropriate transformation parameter.

## VI. CONCLUSIONS

In this paper we have studied the BRST symmetry for the ultraviolet finite, super-renormalizable theory of massless QED<sub>3</sub>. Also we have derived the Nielsen identities for QED<sub>3</sub>. The Nielsen identities are important to investigate because these offer possibilities to check one's calculations as they allow us to see where physical meaning may be found in apparently gauge dependent Green's functions. Further we have generalized the BRST symmetry of the theory by making the transformation parameter finite and field dependent which is known as FFBRST transformation. The fascinating feature of FFBRST transformation is that under change of variables it leads to a non-trivial Jacobian for the path integral measure of generating functional. This Jacobian, written as  $e^{iS_1}$  for some local functional of fields  $S_1$ , depend on the choice of finite field-dependent parameter. We have computed the Jacobian for FFBRST transformation with appropriate finite field-dependent parameter. Remarkably, we have found that the Lowenstein-Zimmermann mass terms together with the external sources for massless QED<sub>3</sub> emerges naturally within functional integral through the Jacobian of a single FFBRST transformation. This is remarkable feature of FFBRST symmetry that any gauge invariant (BRST exact) quantity can be generated through the FFBRST symmetry. Although these Lowenstein-Zimmermann terms are mass terms but are gauge invariant also. Thus we have seen that the extra physical degree's of freedom emerges due to the non-linear BRST transformations (23) where the parameter  $\Theta$  exhibits the extra physical degrees of freedom due to the mass terms. Though we illustrated our results for QED<sub>3</sub> theory but certainly these are not limited to a particular theory. However it is a more general result and can be applied to any gauge theory to get gauge invariant mass terms and there dynamics.

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